Basic Concepts of Fuzzy Logic

Apparatus of fuzzy logic is built on:

- **Fuzzy sets**: describe the value of variables
- **Linguistic variables**: qualitatively and quantitatively described by fuzzy sets
- **Possibility distributions**: constraints on the value of a linguistic variable
- **Fuzzy if-then rules**: a knowledge

Fuzzy sets

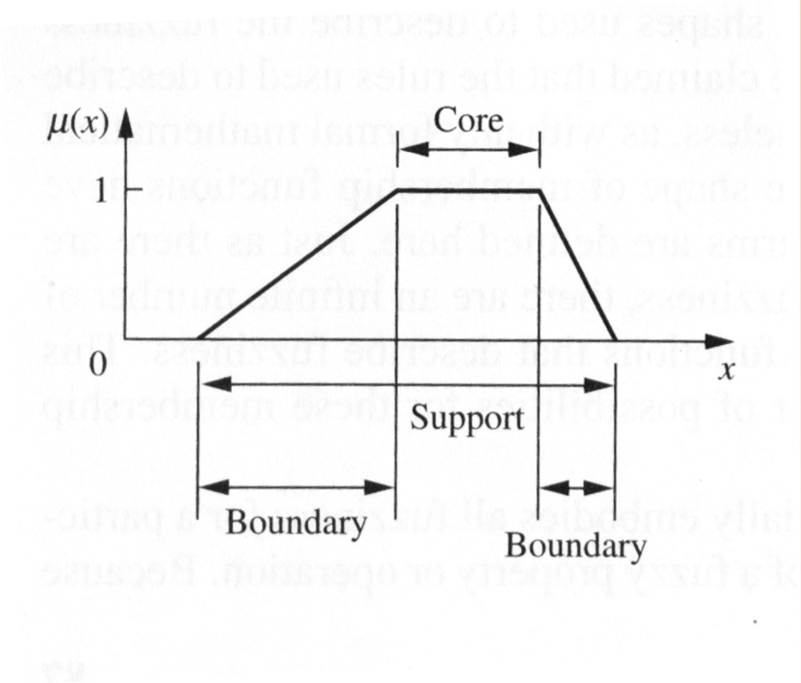
A fuzzy set is a set with a smooth boundary.

A fuzzy set is defined by a functions that maps objects in a domain of concern into their membership value in a set.

Such a function is called the *membership function*.

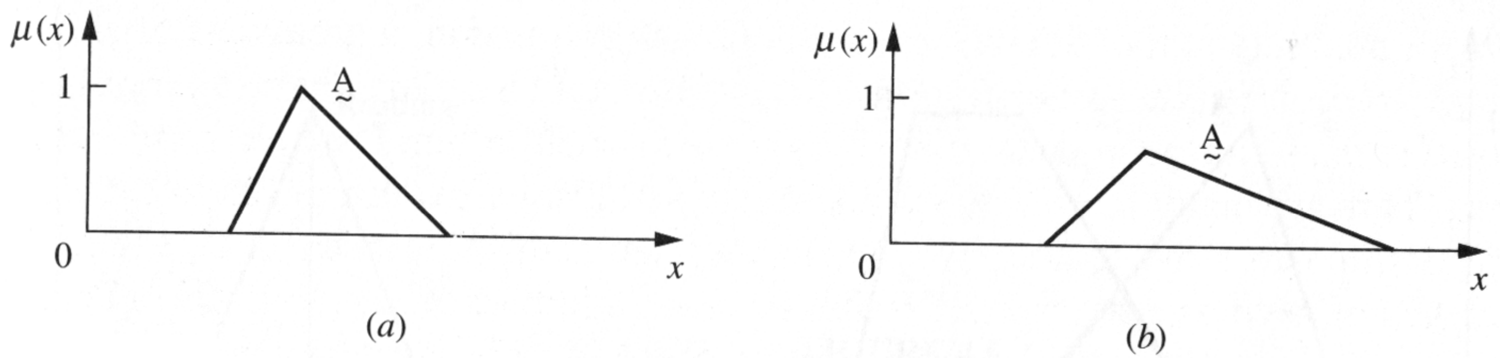
Features of the Membership Function

- **Core:** comprises those elements *x* of the universe such that $\mu_a(x) = 1$.
- **Support :** region of the universe that is characterized by nonzero membership.
- **Boundary :** boundaries comprise those elements *x* of the universe such that $0 < \mu_a(x) < 1$



Features of the Membership Function (Cont.)

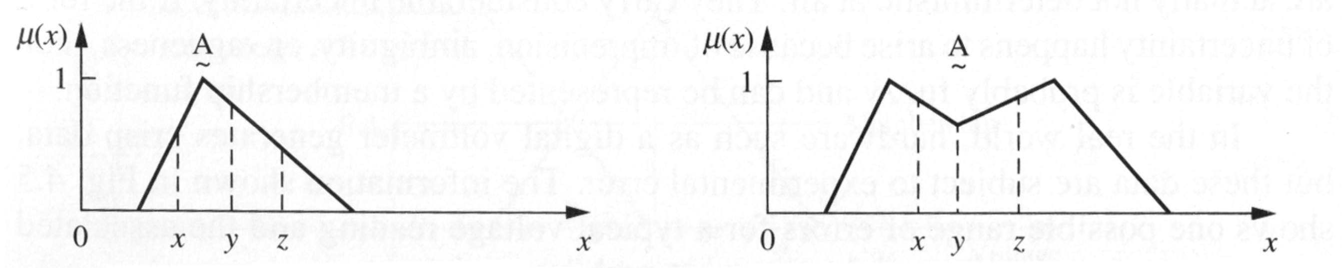
- **Normal Fuzzy Set : at least one element x in the universe whose membership value is unity**



Fuzzy sets that are normal (*a*) and subnormal (*b*).

Features of the Membership Function (Cont.)

- **Convex Fuzzy set:** membership values are strictly monotonically increasing, or strictly monotonically decreasing, or strictly monotonically increasing then strictly monotonically decreasing with increasing values for elements in the universe.



Convex, normal fuzzy set

Non convex, normal fuzzy set

$$\mu_a(y) \geq \min[\mu_a(x), \mu_a(z)]$$

Features of the Membership Function (Cont.)

- **Cross-over points :** $\mu_a(x) = 0.5$

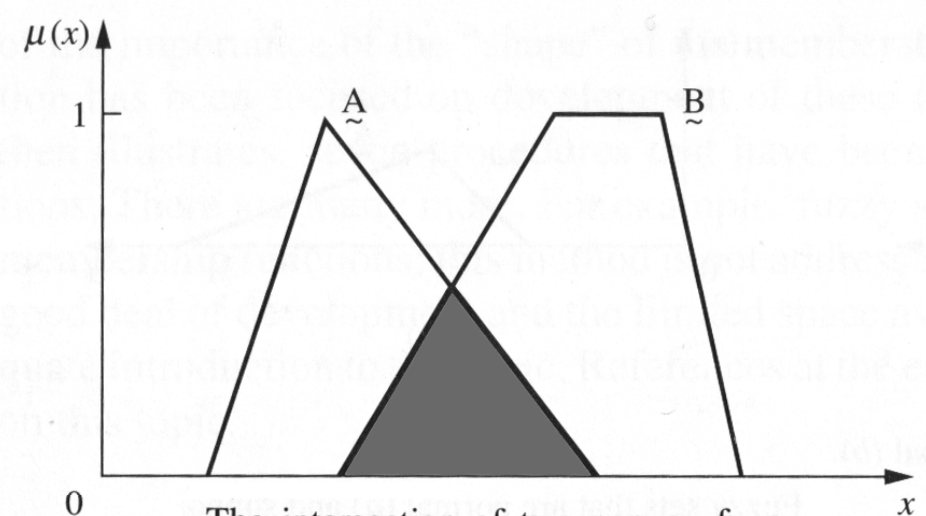
- **Height: defined as max** $\{\mu_a(x)\}$

Operations on Fuzzy Sets

- **Logical connectives:**
  - Union
    - A U B = max(χ_a (x) , χ_b (x))
  - Intersection
    - A . B = min(χ_a (x) , χ_b (x))
  - Complementary
    - A ---> χ_a (x) = 1- χ_a (x)

Features of the Membership Function (Cont.)

- **Special Property of two convex fuzzy set:**
  - for *A* and *B*, which are both convex, *A* . *B* is also convex.



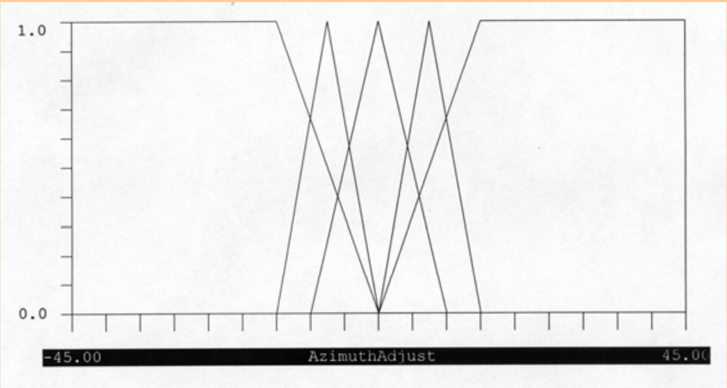
The intersection of two convex fuzzy sets produces a convex fuzzy set.

Design Membership Functions

Manual

- Expert knowledge. Interview those who are familiar with the underlying concepts and later adjust. Tuned through a trial-and-error
- Inference
- Statistical techniques (Rank ordering)

Intutition



- Derived from the capacity of humans to develop membership functions through their own innate intelligence and understanding.
- Involves contextual and semantic knowledge about an issue; it can also involve linguistic truth values about this knowledge.

Inference

- Use knowledge to perform deductive reasoning, i.e . we wish to deduce or infer a conclusion, given a body of facts and knowledge.

Inference : Example

- In the identification of a triangle
  - Let A, B, C be the inner angles of a triangle
    - Where $A \geq B \geq C$
  - Let U be the universe of triangles, i.e.,
    - $U = \{(A,B,C) \mid A \geq B \geq C \geq 0; A+B+C = 180^\circ\}$
  - Let 's define a number of geometric shapes
    - I Approximate isosceles triangle
    - R Approximate right triangle
    - IR Approximate isosceles and right triangle
    - E Approximate equilateral triangle
    - T Other triangles

Inference : Example

- We can infer membership values for all of these triangle types through the method of inference, because we possess knowledge about geometry that helps us to make the membership assignments.

- For Isosceles,
  - $\mu_I(A,B,C) = 1 - 1/60 * \min(A-B, B-C)$
  - If A=B OR B=C THEN $\mu_I(A,B,C) = 1$;
  - If A=120°,B=60°, and C =0° THEN $\mu_I(A,B,C) = 0$.

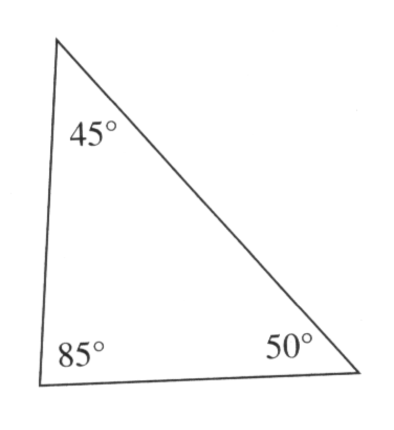
Inference : Example

- For right triangle,
  - μ_R (A,B,C) = 1- 1/90* |A-90°|
  - If A=90° THEN μ_i (A,B,C) = 1;
  - If A=180° THEN μ_i (A,B,C) = 0.
- For isosceles and right triangle
  - IR = min (I, R)
  - μ_{IR} (A,B,C) = min[μ_I (A,B,C), μ_R (A,B,C)]
    = 1 - max[1/60min(A-B, B-C), 1/90|A-90|]

Inference : Example

- For equilateral triangle
  - μ_E (A,B,C) = 1 - 1/180* (A-C)
  - When A = B = C then μ_E (A,B,C) = 1,
    A = 180 then μ_E (A,B,C) = 0
- For all other triangles
  - T = (I.R.E)' = I'.R'.E'
    = min {1 - μ_I (A,B,C) , 1 - μ_R (A,B,C) , 1 - μ_E (A,B,C)

Inference : Example



- Define a specific triangle:
  - A = 85˚ ≥ B = 50˚ ≥ C = 45˚

$\mu_R = 0.94$

$\mu_I = 0.916$

$\mu_{IR} = 0.916$

$\mu_E = 0.7$

$\mu_T = 0.05$

Rank ordering

- Assessing preferences by a single individual, a committee, a poll, and other opinion methods can be used to assign membership values to a fuzzy variable.
- Preference is determined by pairwise comparisons, and these determine the ordering of the membership.

Rank ordering: Example

	Number who preferred							
	Red	Orange	Yellow	Green	Blue	Total	Percentage	Rank order
Red	—	517	525	545	661	2,248	22.5	2
Orange	483	—	841	477	576	2,377	23.8	1
Yellow	475	159	—	534	614	1,782	17.8	4
Green	455	523	466	—	643	2,087	20.9	3
Blue	339	524	386	357	—	1,506	15	5
Total						10,000		

Design Membership Functions

Automatic or Adaptive

- Neural Networks
- Genetic Algorithms
- Inductive reasoning
- Gradient search

Will study these techniques later

Guidelines for membership function design

- Always use parameterizable membership functions. Do not define a membership function point by point.
  - Triangular and Trapezoid membership functions are sufficient for most practical applications!